THE NEUTRINO SEE-SAW IN SO(10)

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When a predictive and apparently realistic model for the quark mass matrices is extended to the leptons by using an SO(10) framework, it is found that the mass matrix of heavy "right-handed" neutrinos is likely to be close to a singular one. This may result in a variety of interesting patterns for the neutrino masses which are analyzed and compared with experiment.

The SO(10) grand unified model [1] provides a traditional guide for thinking about neutrino masses. In the case of three generations the 6×6 neutrino mass matrix M_ν has the form [‡1]

$$M_\nu = \begin{pmatrix} \beta S' & M_{\nu D}^T \\ M_{\nu D} & \gamma S' \end{pmatrix}, \tag{1}$$

where $M_{\nu D}$ and S' are 3×3 matrices whose elements have magnitudes comparable to the charged fermion masses. The dimensionless parameter β (proportional to the vacuum value of an $SU(2)_L$ Higgs triplet) must be very small – say less than 10^{-8} or so – while the dimensionless parameter γ (proportional to an $SU(2)_L$ Higgs singlet) should be very large, say of the order of 10^{13}. Given these values of β and γ it is reasonable to approximate the effective 3×3 matrix for the ordinary light neutrinos (which must be of "majorana" type in general) as

$$M_\nu^{\text{eff}} \simeq \beta S' - \gamma^{-1} M_{\nu D}^T S'^{-1} M_{\nu D} . \tag{2}$$

The γ^{-1} term represents the "see-saw" mechanism [3] and is generally considered to be the dominant one, although this is really a dynamical assumption. The approximation in (2) is justified unless S' is a singular matrix ($\det S' = 0$).

Curiously, in what seems to be good guess for the matrices involved S' may get close to, or actually become, singular. Of course, physicists are (perhaps justifiably) suspicious of statements about the various fermion mass-matrices since they clearly should come from some yet to be discovered deeper physical theory. Nevertheless, the semi-empirical approach to mass levels has an honorable history. Recently it was pointed out [4,5] that combining two Ansatze – that of Fritszch [6] and that of Stech [7] – which had been at first glance considered incompatible results in the possibility of predicting (for any number of generations) the Kobayashi–Maskawa matrix completely in terms of the quark masses. In the case of three generations the predictions are in agreement with our present experimental information – they also feature a "maximal" *CP* violation phase and require 25 GeV $< m_{\text{top}} <$ 45 GeV. A natural way to extend the model to leptons is to embed it in SO(10). This has been suggested by Bottino, Kim, Nishiura and Sze [8] and we report here on our further study along that line. In this model the mass matrices are

$$M_u = S + \epsilon S' , \quad M_d = \alpha M_u + A + (1 - \alpha\epsilon) S' , \quad r M_e = \alpha M_u + \delta A - (3 + \alpha\epsilon) S' , \tag{3a,b,c}$$

for the charge 2/3 quarks, charge –1/3 quarks and charge –1 leptons, respectively. S and S' are symmetric ma-

[‡1] A review of neutrino masses in the SO(10) theory is given in ref. [2].

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trices describing the coupling patterns of the **10** and **126** Higgs fields while A is an antisymmetric matrix describing the couplings of the **120** Higgs field. α, δ and ϵ are dimensionless real constants very roughly of order unity. Finally, r is a renormalization factor appropriate to comparing the quark masses with the charged lepton masses at a low energy scale (say 1 GeV) rather than at the normal unified scale. It is traditionally [9] taken to be about 3 but, considering the uncertainties involved in its determination, one would be unhappy if any physical results were to change drastically as r varied over the region

$$3 < r < 4 . \tag{4}$$

In this model the neutrino masses are given by (1) with S' determined from eqs. (3) and with the reasonable approximation

$$r' M_{\nu D} \simeq S . \tag{5}$$

r' is a renormalization group parameter which should be a little larger than r (say $r' = 3.5$ if $r = 3.0$). So far essentially SO(10) with one Higgs field of each relevant type has been assumed. The model is completed by requiring each matrix S, S' and A to be hermitian and the mass matrices to have the form, for example

$$r M_e = Q \begin{pmatrix} 0 & A_e & 0 \\ A_e & 0 & B_e \\ 0 & B_e & C_e \end{pmatrix} Q^{-1} , \quad Q = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}) , \tag{6}$$

where

$$A_e = (m_e m_\mu m_\tau / C_e)^{1/2} , \quad B_e = [(m_\tau + m_e)(m_\tau - m_\mu)(m_\mu - m_e)/C_e]^{1/2} , \quad C_e = m_\tau - m_\mu + m_e .$$

A similar form holds for M_d and (without any phases) for M_u. Matrices of the form (6), though not symmetric in general, give the same predictions as the Fritzsch model. Notice that the choice $\alpha\epsilon = 1$ gives

$$M_d = M_d^\dagger = \alpha M_u + A , \quad M_u = M_u^T = M_u^* , \tag{7}$$

which is the original Stech model [7]. We shall adopt this choice here for convenience. If we relax $\alpha\epsilon$ from unity we can accomodate a top quark mass as high as about 95 GeV but the predictions of the model are otherwise very similar.

The possible singularity of S' does not depend on any fine details of the model, from (3b) and (3c) we get without any assumptions

$$\mathrm{Tr}\, S' = S'_{33} = (\mathrm{Tr} M_d - r\, \mathrm{Tr} M_e)/4 = [(m_b - m_s + m_d) - r(m_\tau - m_\mu + m_e)]/4 , \tag{8}$$

where the Fritzsch type form (6) was used in the first step. Since $\det S' = -S'_{33}(S'_{12})^2$ we see that S' becomes singular when r takes the "magic" value

$$r = \frac{m_b - m_s + m_d}{m_\tau - m_\mu + m_e} = 3.0\text{–}3.5 , \tag{9}$$

depending on the precise values [10] of the running masses, m_s (1 GeV) and m_b (1 GeV). Comparing (9) with (4) shows that the singularity could occur in the expected region for r. Notice that the above argument for the possible singularity of S' would hold if there were any number of **10**'s and/or **120**'s present, assuming the coupling matrices to have the Fritzsch form. As we shall see the width of the "magic canyon" (region where the effects of the singularity are important) is very small for the see-saw term in (2) (due to the large value of γ) so one could always avoid it with a clear conscience. However the fact that it occurs near the expected value of r suggests that Nature may be trying to tell us something. If the system were exactly at a singular point there would be only two superlight Majorana neutrinos and only two superheavy neutrinos. The other two (presumably related to the τ neutrino) would coalesce to form a Dirac neutrino of mass in the range of tens to hundreds of MeV. Since the experimental upper bound [11] [‡2] on $m(\nu_\tau)$ is as large as 50–85 MeV, such a picture is not a priori unreasonable.

‡2 Ref. [11] refers to the earlier less restrictive bounds.

In order to study the neutrino masses and mixings for the singular case we must give up the approximation (2) and return to the original equation (1). At the singularity the effect of β is not qualitatively important so we might as well set $\beta = 0$. With (5) and the notations

$$S = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}, \quad S' = \begin{pmatrix} 0 & a' & 0 \\ a' & 0 & b' \\ 0 & b' & c' \end{pmatrix}, \tag{10}$$

the secular equation for (1) then becomes

$$\lambda^6 + \sum_{n=0}^{5} d_n \lambda^n = 0 , \tag{11}$$

where

$$d_0 = -a^4c^2 , \quad d_1 = 2abcx - c'a^2(a^2+b^2) ,$$
$$d_2 = (a^2+b^2)^2 + x^2 + c^2(a'^2+2a^2) , \quad d_3 = -2bcb' + c'(a'^2+2a^2+b^2) ,$$
$$d_4 = -2a^2 - 2b^2 - c^2 - a'^2 - b'^2 , \quad d_5 = -c' , \quad x = ab' - ba' .$$

Remember that the primed quantities are larger in order of magnitude than the unprimed ones by the huge factor of γ. A singularity of S' is achieved if $c' \equiv S'_{33}$ is zero which corresponds to r taking on the magic value given in (9). When $c' = 0$ the equation determining the superheavy neutrino masses of order γ only involves the λ^6 and λ^4 terms and we find them to be [accurate to order $(1/\gamma)$]

$$\lambda_{5,6} = \pm(a'^2 + b'^2)^{1/2} . \tag{12}$$

We see that the two superheavies have the same mass (of course, a sign is irrelevant) so they can be considered as a single Dirac neutrino. If $c' \neq 0$ the equation determining the superlight neutrinos (of order γ^{-1}) would be a cubic and we would get three of them. However, when $c' = 0$, only the λ^2, λ^1, and λ^0 terms in (11) are relevant and we find only two superlights with masses

$$\lambda_{1,2} = [xac/(a'^2c^2 + x^2)]\,\{-b \pm [b^2 + a^2(1 + a'^2c^2/x^2)]^{1/2}\} . \tag{13}$$

Finally, noting that $\det M_\nu = \Pi_{i=1}^{6}\lambda_i = -a^4c^2$ we get from (12) and (13) that

$$-\lambda_3\lambda_4 = \lambda_3^2 = (x^2 + c^2a'^2)/(a'^2 + b'^2) , \tag{14}$$

where we also used $\lambda_4 \simeq -\lambda_4$ which holds to excellent accuracy since $\Sigma_{i=1}^{6}\lambda_i = \mathrm{Tr} M_\nu = 0$. Eq. (14) shows that, as mentioned before, the putative τ-neutrino here is of Dirac type and has a mass of the same order as the charged lepton masses. Notice that this mass is independent of γ. The expression (14) is rougly minimized by setting $a' = 0$ which yields

$$\lambda_3 \simeq a \simeq (m_u m_c)^{1/2}/r' \simeq 24 \text{ MeV} . \tag{15}$$

This seems promising for a realistic but exotic τ-neutrino. However, the approximate third and fourth eigenvectors of M_ν (obtained by successively first diagonalizing the lower right 3×3 submatrix of M_ν and then the upper left 4×4 submatrix) are

$$\psi_{3,4} \simeq [2(x^2 + a'^2c^2)]^{1/2} \begin{pmatrix} 0 \\ x \\ -ca' \\ \pm(x^2 + a'^2c^2)^{1/2} \\ 0 \\ 0 \end{pmatrix} . \tag{16}$$

For $a' = 0$ our 24 MeV neutrino is mostly along the mu rather than the tau direction (the effect of diagonalizing the charged lepton mass matrix turns out not to modify this feature much) which is unrealistic. An evident way out is to consider $x = 0$. Then $\lambda_3 \simeq ca/r'b \simeq (m_u m_t)^{1/2}/r' \simeq 150$ MeV, which is a little too large for the present bound $m(\nu_\tau) < 50$–85 MeV. Thus, to achieve a realistic theory, some detuning from magic r may be indicated. This is best handled by a numerical rather than an analytical approach.

We have carried out a detailed numerical analysis of this model which will be reported elsewhere [12]. If the masses of the nine charged fermions were *precisely* known, all parameters of the mass matrices M_u, M_d and M_e would be fixed in the $\alpha\epsilon = 1$ case, except for one in the lepton sector, which may be taken to be δ (a measure of *CP* violation in M_e). Fixing δ would then enable us to completely predict the neutrino masses and mixing matrix for given choices of the parameters β and γ in (1). In practice, the fact that the quark masses and the Kobayashi–Maskawa mixing angles $|U_{us}|$ and $|U_{cb}|$ are known only to a certain accuracy must be taken into account to find the allowed regions in parameter space. For definiteness, we will here simply make the following consistent choices for the quark masses:

$$m_u/m_d = 0.57\,, \quad m_d/m_s = 0.056\,, \quad m_s = 0.16\,\mathrm{GeV}\,, \quad m_c = 1.35\,\mathrm{GeV}, \quad m_b = 5.3\,\mathrm{GeV}\,, \quad m_t = 53\,\mathrm{GeV}\,. \tag{17}$$

Let us illustrate a typical scenario for neutrino masses and mixings when r is in the magic canyon but not at the precise singular point. With (17), the magic value is $r \simeq 3.065707$. Choosing for simplicity $\delta = 0$ (no leptonic *CP* violation) and $\gamma = 10^{10}$ we detune slightly (since $m_3 = m_4 = 460$ MeV at magic r here) to $r \simeq 3.065700$ and find from a numerical diagnalization of (1) in our model that the two intermediate mass neutrinos have split to $m_3 = 7.4$ MeV and $m_4 = 28$ GeV. The two superlight neutrino masses are $m_1 = 0.0018$ eV and $m_2 = 0.56$ eV. Notice that the mixing matrix K – which is a 3 × 6 rectangular matrix [13] defined by

$$L = (ig/\sqrt{2}) W_\mu^- \bar{e}_L \gamma_\mu K \nu_L + \text{h.c.}\,,$$

in a standard notation – is now completely predicted and turns out to be

$$K \simeq \begin{pmatrix} 1.00 & \cdot & 0.012i & 0.0098i & 1.6\times10^{-4} \\ 0.014i & & 0.99 & 0.13 & -2.1\times10^{-3}i \\ 0.0082i & & -0.13 & 0.99 & 0.016i \end{pmatrix}. \tag{18}$$

Here we have introduced some factors of i in order to make all neutrino masses positive and have approximated the 3 × 6 by a 3 × 4, neglecting the two superheavies. Let us focus on ν_3 and ask whether the relevant particle-physics and astrophysical bounds [‡3] are satisfied. The non-observation of a ν_3 around 10 MeV in the $\pi^- \to e^- \nu_3$ and $\pi^- \to \mu^- \nu_3$ reactions [15] results in the bounds $|K_{13}| < 0.01$ and $|K_{23}| < 0.15$ which are just consistent with (18). Astrophysical bounds [14] require ν_3 to decay rapidly. Since $m(\nu_3) > 2m(e^-)$ here, the quick decay channel $\nu_3 \to e^- e^+ \nu_e$ is open and one needs roughly $|K_{13}| > 0.005$ which is again consistent. However, the non-observation of neutrinoless double beta decay requires [‡4]

$$\left| \sum_{i=1}^{3} m_i (K_{1i})^2 \right| \approx m_3 |(K_{13})^2| < 10\ \mathrm{eV} \quad \text{or} \quad |K_{13}| < 0.001\,,$$

which is not satisfied. It is amusing, though, that precisely at magic r the contributions of ν_3 and ν_4 to neutrinoless double beta decay cancel each other. The splitting of m_3 and m_4 destroys this cancellation. A number of options remain to construct a viable theory by complicating the model. Our main point here has been to show the existence of a new scenario for the neutrino mass spectrum. Thus one should not a priori rule out a tau neutrino mass as large as 10 MeV. A one-order-of-magnitude improvement in the measurement of the tau neutrino mass just might reveal an interesting surprise.

‡3 A summary is given in ref. [14]. See the diagram on page 107.
‡4 A review is given in ref. [16].

Since this model is very predictive it is still quite interesting even if we imagine that r is well out of the magic canyon. Then we may consider two obvious scenarios – the dominance of the β term in (2) or the dominance of the γ term. The case where the β term is dominant was discussed in ref. [8] where it was found that the results were pretty much independent of the input parameters. This can be easily understood since S' is close to the singular matrix

$$M_\nu^{\mathrm{eff}} \simeq \beta S' \simeq \beta \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & S'_{23} \\ 0 & S'_{23} & 0 \end{pmatrix}.$$

Note that the closeness to the singularity will be important for the β term even if r differs from its magic value by as much as 0.5 or so. The singular limit has a zero-mass first neutrino and two degenerate heavier neutrinos which mix among themselves. This pattern is only slightly modified by the departure from the singularity. For example with $r = 3.00$ and $\delta = 0.2$ we find

$$(m_1, m_2, m_3) = (10^9 \beta) \times (2.2 \times 10^{-5}, 0.10, 0.13)\ \mathrm{eV},$$

and the mixing matrix

$$K = \begin{pmatrix} 1.00 & 0.042@-62^\circ & 0.062@-169^\circ \\ 0.07@-110^\circ & 0.82 & 0.57@-91^\circ \\ 0.029@1^\circ & 0.57@-89^\circ & 0.82 \end{pmatrix}, \tag{19}$$

where $\rho@\theta \equiv \rho e^{i\theta}$.

This model features substantial $\nu_\mu - \nu_\tau$ mixing. It is interesting that an experiment on $\nu_\mu - \nu_\tau$ oscillations [17] then gives a strong bound on β. From the condition that $(m_3)^2 - (m_2)^2 < 5\ \mathrm{eV}^2$ we obtain $\beta < 2.5 \times 10^{-8}$ so that the heaviest neutrino should weigh less than about 3 eV.

The case when the second (see-saw) term of eq. (2) is dominant is the most orthodox possibility. With the same parameters as above and $\gamma = 10^{12}$ we find the light neutrino masses

$$(m_1, m_2, m_3) = (1.94 \times 10^{-5}, 0.0059, 7.90)\ \mathrm{eV},$$

and the mixing matrix

$$K = \begin{pmatrix} 1.00 & 0.0240@161^\circ & 0.010@-66^\circ \\ 0.0247@21^\circ & 0.99 & 0.133@185^\circ \\ 0.0086@-97^\circ & 0.133@-5^\circ & 0.99 \end{pmatrix}. \tag{20}$$

This example is relevant for a recent model [18] which explains the lack of solar ν_e's observed on earth as resulting from the transformation of ν_e's produced at high density in the center of the sun into ν_μ's. For this purpose one may have [19] m_1 negligible and $m_2 \simeq 0.007$ eV in agreement with this example. Furthermore it is necessary for the $\nu_e \to \nu_\mu$ transition to be adiabatic which implies $|K_{12}| > 0.007$. This criterion is also seen to be satisfied in our model. Actually the other two scenarios could (with suitably scaled γ or β) also solve the solar neutrino problem in this way.

We have seen that a plausible model for the mass matrices in SO(10) results in an interesting possible singularity in the "right-handed" neutrino mass matrix which plays a crucial role in the see-saw mechanism. This singularity occurs if, at the grand unified scale, the relation $m_b - m_s + m_d = m_\tau - m_\mu + m_e$ holds, which is very close to the usual expectation $m_b \simeq m_\tau$. A signature for the system lying within the range of influence of this singularity would be a tau neutrino mass of the order of several MeV or so. Even if the system is outside this "magic canyon" the model is very predictive. For example, the recent proposed solution of the solar neutrino problem requires knowledge of the $\nu_e - \nu_\mu$ mixing angle which is predicted here.

A more detailed discussion of this model will be given elsewhere [12].

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